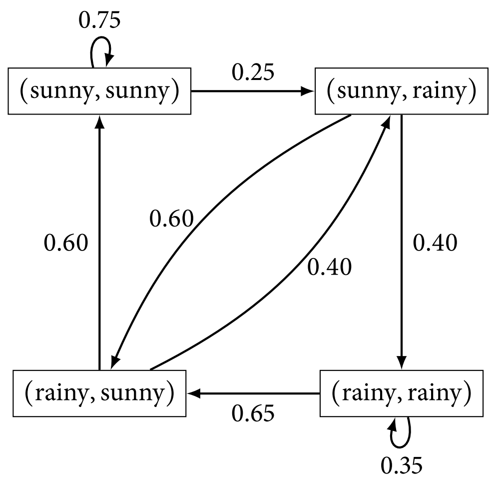
Solutions to Problem 1. For the Markov property to hold, the next toothpaste brand that a customer purchases must only depend on the customer's current toothpaste brand. This may not be reasonable, since brand loyalty usually goes beyond one tube.

For time stationarity to hold, the probabilities of moving between toothpaste brands must be constant over time. This also may not be reasonable, depending on the time horizon, since tastes tend to change over time.

Solutions to Problem 2.

- **State space.** $\mathcal{M} = \{(\text{sunny}, \text{sunny}), (\text{sunny}, \text{rainy}), (\text{rainy}, \text{sunny}), (\text{rainy}, \text{rainy})\}$
- **Time step.** 1 day
- **Transition probabilities.**



Solutions to Problem 3.

- **State space.** $\mathcal{M} = \{(J,1), (J,2), (J,3+), (S,1), (S,2), (S,3+), L\}$
- **Time step.** 1 year
- **Transition probabilities.**

$$\mathbf{P} = \begin{array}{c} \begin{array}{ccccccc} (J,1) & (J,2) & (J,3+) & (S,1) & (S,2) & (S,3+) & L \end{array} \\ \begin{bmatrix} 0 & 0.8 & 0 & 0.1 & 0 & 0 & 0.1 \\ 0 & 0 & 0.8 & 0.1 & 0 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0.4 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.6 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0.1 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ (J,1) \\ (J,2) \\ (J,3+) \\ (S,1) \\ (S,2) \\ (S,3+) \\ L \end{array}$$

- **Initial state probabilities.** $q_{(J,1)} = 1, q_{(J,2)} = q_{(J,3+)} = q_{(S,1)} = q_{(S,2)} = q_{(S,3+)} = q_L = 0$

Solutions to Problem 4.

- **State space.** $\mathcal{M} = \{1, 2, 3\}$. State 1 corresponds to the component being good and declared good. State 2 corresponds to the component being defective but declared good. State 3 corresponds to the component being defective and declared defective.

- **Time step.** 1 item

- **Transition probabilities.**

$$\mathbf{P} = \begin{bmatrix} 0.995 & 0.005(0.06) & 0.005(0.94) \\ 0.495 & 0.505(0.06) & 0.505(0.94) \\ 0.495 & 0.505(0.06) & 0.505(0.95) \end{bmatrix}$$